INCORPORATION OF RANDOM WAVE EFFECTS INTO A QUASI-3D NEARSHORE CIRCULATION MODEL

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Abstract

A coupled wave-hydrodynamic modeling system, comprised of a random wave model driving a quasi-3D nearshore hydrodynamic model, is described. Random wave formulations for several inputs to the hydrodynamic model are developed. An alternate wave dissipation mechanism is incorporated into the random wave model, and two wave roller descriptions are implemented to calculate volume flux and other roller-dependent input properties. Comparison to laboratory and field data indicate that an evolving roller description, in conjunction with the 3D dispersive mixing inherent in the hydrodynamic model, yield the best results. A method to nest the model system inside larger-scale wave models is described, and an application to an area of complex bathymetry shown.

INTRODUCTION

Numerical modeling of wave-induced nearshore circulation has undergone much development in recent years, to the extent that use in field situations is almost routine. The one-dimensional Navy Standard Surf Model (Earle 1989) has been used for forecasting nearshore conditions for Navy operations for several years. The Delft3D model (http://www.wldelft.nl/soft/d3d) is a commercial package used in engineering

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14. ABSTRACT
A coupled wave-hydrodynamic modeling system, comprised of a random wave model driving a quasi-3D nearshore hydrodynamic model, is described. Random wave formulations for several inputs to the hydrodynamic model are developed. An alternate wave dissipation mechanism is incorporated into the random wave model, and two wave roller descriptions are implemented to calculate volume flux and other roller-dependent input properties. Comparison to laboratory and field data indicate that an evolving roller description, in conjunction with the 3D dispersive mixing inherent in the hydrodynamic model, yield the best results. A method to nest the model system inside larger-scale wave models is described, and an application to an area of complex bathymetry shown.

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applications on a worldwide basis. More recently, the quasi-3D model SHORECIRC (Van Dongeren and Svendsen 2000) has been validated with data from the Duck94 experiment (Svendsen et al. 1997). It is therefore opportune to investigate incorporation of wave forcing more in line with general nearshore conditions in the field.

In this paper we discuss the incorporation of random wave effects into the quasi-3D model SHORECIRC. Required wave forcing for the model extends past the standard gradients of radiation stress; random wave extensions of these inputs are developed herein. Comparisons to data (laboratory and field) are shown, and the incorporation of the revised model into a model nesting scheme is described.

SHORECIRC and REF/DIF-S

The SHORECIRC model (Van Dongeren and Svendsen 2000) combines the physical effects of a non-depth-uniform current structure with a two-dimensional modeling formalism, thereby requiring only horizontal discretization of the domain. Semi-analytic solutions are used to replicate the depth structure of the currents, which is then integrated to provide the necessary coefficients for the current-current and current-wave interaction terms in the two-dimensional horizontal model. This allows for the effect of 3D dispersive mixing (Putrevu and Svendsen 1999), essential for obtaining the proper horizonal momentum exchange in the nearshore without requiring unrealistic values of eddy viscosity coefficients for sufficient lateral mixing.

The scales of motion simulated by SHORECIRC are assumed to be averaged over individual waves (wave group scale) and thus wave information is required from a separate model. This information includes gradients of radiation stress; short wave volume flux; near-bed orbital velocities; and estimates of breaking wave dissipation. Short wave volume flux is required due to the quasi-3D nature of the model and the need to balance the shoreward mass flux with the resulting undertow.

The default version of the SHORECIRC model presently available uses the single frequency short wave model REF/DIF-1 (Kirby and Dalrymple 1994) as a wave driver. The REF/DIF-1 model uses the parabolic approximation to the mild-slope equation (Berkhoff 1972; Radder 1979) for wave propagation over varying bathymetry, and is thus capable of simulation both refraction and diffraction, with higher order terms included to enhance model accuracy at oblique angles (Kirby 1986). While useful for development purposes and research applications, general utility is hampered with a monochromatic wave driver. The model REF/DIF-S (Chawla et al. 1998) is a spectral version of REF/DIF-1; it propagates many frequency and direction components over the domain simultaneously within the REF/DIF-1 modeling format, using the dissipation function of Thornton and Guza (1983) to represent energy decay in the spectrum due to breaking. Both REF/DIF-1 and REF/DIF-S are wave-resolving models, and thus directional spectra must be decomposed into individual wavetrains with random phases for initialization. For later reference, we write the REF/DIF-S governing equation in generic form as:

$$C_{gn} A_n + M A_{nyy} + N A_n + P A_{nxy} + Q A_{nyyx} + \alpha A_n = 0 \tag{1}$$

where C_{gn} is the group velocity for wave component n; M, N, P and Q are coefficients for various transformation processes representing wave shoaling, refraction, diffraction, wave-current interaction and wide-angle propagation effects; A_n is the complex amplitude of the n'th wave component in the spectrum; and α is the dissipation function used to simulate wave energy decay due to breaking. Subscripts x, y refer to partial differentiation.

WAVE FORCING

The REF/DIF-1 model used in the default version of SHORECIRC provides the hydrodynamic model with all necessary forcing components. It uses the results of Svendsen (1984) to calculate short wave volume flux inside and outside the surf zone, and uses Dally et al. (1985) to provide dissipation estimates. Since REF/DIF-1 is monochromatic, a demarcation between non-breaking and breaking regions is possible, and surf zone mechanisms can be switched on or off accordingly. This division is not possible in irregular wave models such as REF/DIF-S. While the probabilistic dissipation function of Thornton and Guza (1983), included in the default version of REF/DIF-S, works well for simulating waveheight decay, there is no clear methodology for incorporating or excluding surf zone processes due to this lack of a "breakpoint;" this problem needs to be addressed. Additionally, random wave equivalents for several input quantities such as short wave volume flux require development.

Dissipation Mechanisms

The default dissipation mechanism in REF/DIF-S is that of Thornton and Guza (1983). This function was adapted for complex amplitude models by Chawla et al. (1998):

$$\alpha = \frac{3\sqrt{\pi}}{4} \frac{f_p B^3}{\gamma^4 h^5} H_{rms}^5 \tag{2}$$

where B and γ are free parameters and f_p is the peak frequency. The dissipation function of Battjes and Janssen (1978) was added to REF/DIF-S as an alternative. It was adapted for complex amplitude models by Eldeberky and Battjes (1996):

$$\alpha = \frac{1}{4} \frac{f_p Q_b H_{max}^2}{\sum C_{gn} |A_n|^2} \tag{3}$$

where Q_b is the percentage of breaking waves, calculated by the following implicit relationship:

$$\left(\frac{1 - Q_b}{\ln Q_b} \right) = - \left(\frac{H_{rms}}{H_{max}} \right)^2 \tag{4}$$

$$H_{max} = \frac{0.88}{\overline{k}} \tanh \left(\frac{\gamma' \overline{k} h}{0.88} \right) \tag{5}$$

where γ' is a parameter dependent on offshore wave steepness, and $\overline{k}$ is the wavenumber associated with the characteristic frequency for the spectrum (usually the peak frequency). Either dissipation function is a selectable option in the model.

Roller Descriptions

In order to calculate the forcing and short wave volume fluxes, we will require expressions for the roller area (per unit length of crest) A' and the roller energy density E_r. These are in turn dependent on the particular description of the roller characteristics selected.

In REF/DIF-S we make use of two roller descriptions. The first is denoted the "static roller" because it is dependent entirely on local water depth properties. We use the result of Lippmann et al. (1996), who extended the roller area expression for weak hydraulic jumps to include random wave effects by integrating the cube of the breaking waveheight through the probability density function for breaking waves (Thornton and Guza 1983). The resultant expression for roller area A' is:

$$A' = \frac{(B\langle H_b\rangle)^3}{4h\tan\sigma} \tag{6}$$

where:

$$\langle H_b^3\rangle = \frac{3\sqrt{\pi}}{4}\frac{1}{(\gamma h)^2}H_{rms}^5\left(1 - \frac{1}{\left(1+\left(\frac{H_{rms}}{\gamma h}\right)^2\right)^{\frac{5}{2}}}\right) \tag{7}$$

and σ is the angle of the stress vector at the wave/roller interface. The roller energy density E_r is found from (Svendsen 1984):

$$E_r = \frac{\rho A'\overline{C}}{2\overline{L}} \tag{8}$$

where $\overline{C}$ is the average phase speed and $\overline{L}$ is the average wavelength.

The second description uses the evolving roller model of Stive and deVriend (1994). The governing equation for the evolving roller is:

$$\frac{d(2E_r\overline{C}\cos\overline{\theta})}{dx} = \overline{S} - \frac{2gE_r\sin\beta}{\overline{C}} \tag{9}$$

where $\overline{S}$ is the dissipation from wave breaking (using the original form of dissipation prior to adaptation for REF/DIF-S) and β is the slope of the roller interface. This model is solved using a centered finite difference scheme solved at every step of the REF/DIF-S model to yield E_r through the domain. The model is initialized with $E_r = 0$ at the offshore boundary. The roller area A' is then calculated from E_r using (8).

Radiation Stress in the Surf Zone

The radiation stress evaluations in REF/DIF-S essentially sum contributions to the overall radiation stress from each frequency and direction component in the spectrum. Inside the surf zone, however, a randomization of the expression of Svendsen (1984) for radiation stress due to rollers is used:

$$S_{xxr} = 2E_r\cos^2\overline{\theta} \tag{10}$$

$$S_{yyr} = 2E_r \sin^2 \overline{\theta} \tag{11}$$

$$S_{xyr} = 2E_r \cos \overline{\theta} \sin \overline{\theta} \tag{12}$$

where $\overline{\theta}$ is the mean angle. This is then added to the radiation stress contributions from the non-breaking waves.

Short Wave Volume Flux

Outside the surf zone, short wave volume flux is essentially the Stokes drift. Within the surf zone, the form of the volume flux depends on the assumed characteristics of the breaking wave. For monochromatic waves, Svendsen (1984) determined a volume flux assuming that the breaking wave can be described by a “roller” of white water rotating on the face of an unbroken wave. For random waves, we assume that the short wave volume flux can be described generically as:

$$Q_{wrx} = \frac{A'}{\overline{T}} \cos \overline{\theta} \tag{13}$$

$$Q_{wry} = \frac{A'}{\overline{T}} \sin \overline{\theta} \tag{14}$$

where $\overline{T}$ is the average wave period. This is then added to the Stokes drift to obtain the total short wave volume flux.

Eddy viscosity specification

As mentioned previously, the quasi-3D nature of the SHORECIRC allows proper mixing to take place without requiring egregiously large values of eddy viscosity coefficient. Thus the turbulent shear stresses contribute litte to the overall mixing. The eddy viscosity model used for the turbulent shear stresses is depth uniform; the coefficient is:

$$\nu_t = C_1 \kappa \sqrt{\frac{f_w}{2} u_o h} + Mh \left(\frac{\alpha}{h} \right)^{1/3} + \nu_{to} + \nu_s \tag{15}$$

where κ is von Karman’s constant (≈ 0.4); C_1 and M are coefficients for bottom-induced turbulence and breaking turbulence, respectively; f_w is the wave-related bottom friction coefficient; u_o is the magnitude of the near-bed particle velocity; and ν_{to} and ν_s are the background and Smagorinsky subgrid viscosities, respectively. While the first term of (15) is relevant for the entire domain, the second term only pertains to the surf zone. Since the dissipation mechanism is always activated in REF/DIF-S, all terms in (15) are active for the entire domain, with the dissipation α weighting the contribution of the breaking wave turbulence. Additionally an estimate of $u_{1/3}$ replaces u_o in REF/DIF-S.

COMPARISON TO DATA

In this section we compare the results of SHORECIRC from the different forcing

mechanisms to data. Our first comparison is to the data of Reniers and Battjes (1997), who ran both monochromatic and irregular waves over a sloping bathymetry with a nearshore bar. Waveheights, mean sea surface and nearshore depth-averaged currents were measured. They compared their results to a one-dimensional wave-current model, with good results.

Here we compare the SHORECIRC model driven with both the Thornton and Guza (1983) and Battjes and Janssen (1978) dissipation models. Both the static roller and the Stive and deVriend (1994) roller are used. Figure 1 shows a comparison of modeled currents to data. The modeled currents resulted from the following combinations: Battjes and Janssen dissipation with Stive and deVriend roller; Thornton and Guza dissipation with Stive and deVriend roller; and Thronton and Guza dissipation with static roller. All tests were performed with the quasi-3D dispersive mixing mechanism activated. Free parameters for the Thornton and Guza decay were $B = 0.8$ and $\gamma = 0.6$. For the static roller $\sigma = 10^o$, while for the evolving roller $\beta = 0.1$. The bottom friction coefficient $f_w = 0.02$, while the coefficients for (15) are $C_1\kappa = 0.08$ and $M = 0.08$. Both dissipation mechanisms performed well in capturing the waveheight decay, with the Thornton and Guza result exhibiting a slightly better comparison. However, the Battjes and Janssen decay with the Stive and deVriend roller yielded the most favorable comparison to velocity data; the Thornton and Guza decay with the Stive and deVriend roller also compared reasonably well. On the other hand, the static roller resulted in an unrealistically narrow longshore current profile; it is apparent that the evolving roller of Stive and deVriend (1994) yields a more realistic longshore current profile. Tests with the quasi-3D mechanism deactivated (not shown) clearly exhibited insufficient mixing, even with increased values of eddy viscosity coefficient.

Comparisons to field data from the Duck94 experiment (held at Duck, NC during October 1994) were performed to evaluate the model under field conditions. Nearshore waveheights, undertow profiles and depth averaged currents were compared to measurements for the case of October 12, 1994. Both a high-tide condition (12:30 EST) and a mid-tide condition (16:20 EST) were simulated with the REF/DIF-S-forced SHORECIRC model, using the Battjes and Janssen decay with the Stive and deVriend roller. Figures 2 and 3 show the high tide case. From Figure 2 it is apparent that the depth profiles for the longshore currents are well simulated. The cross-shore current comparison does evidence some deviation from measurements over the bar near the surface. Figure 3 shows comparisons to the nearshore waveheights and depth-averaged longshore currents. The model appears to underpredict the amount of dissipation in the wavefield. This insufficient dissipation over the bar is coincident with the underprediction of the undertow curvature and the maximum longshore current. This is presently being investigated; however, the level of agreement displayed here is encouraging.

MODEL NESTING

To fiurther increase the general utility of the REF-DIF/S-SHORECIRC modeling system, a provision for accepting initial and boundary conditions from offshore models is included. Generally, initial conditions for nearshore wave models are provided by

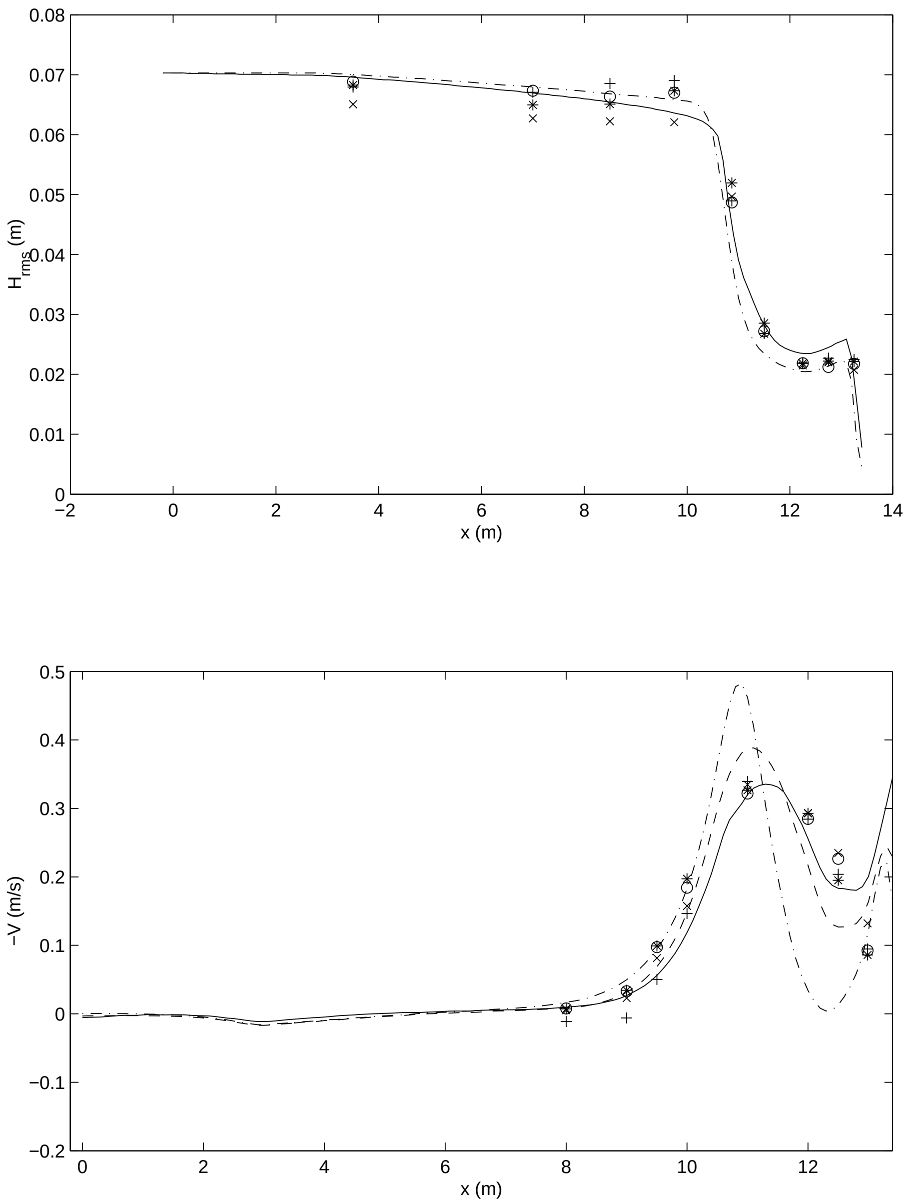

Figure 1: Comparison to experiment of Reniers and Battjes (1997). Top: H_{rms}; solid line: Battjes and Janssen decay; dashed line: Thornton and Guza decay. Bottom: Longshore current; solid line: Battjes and Janssen decay with Stive and deVriend roller; dashed line: Thornton and Guza decay with Stive and deVriend roller; dashed-dot line: Thornton and Guza decay with static roller. Symbols indicate measurements.

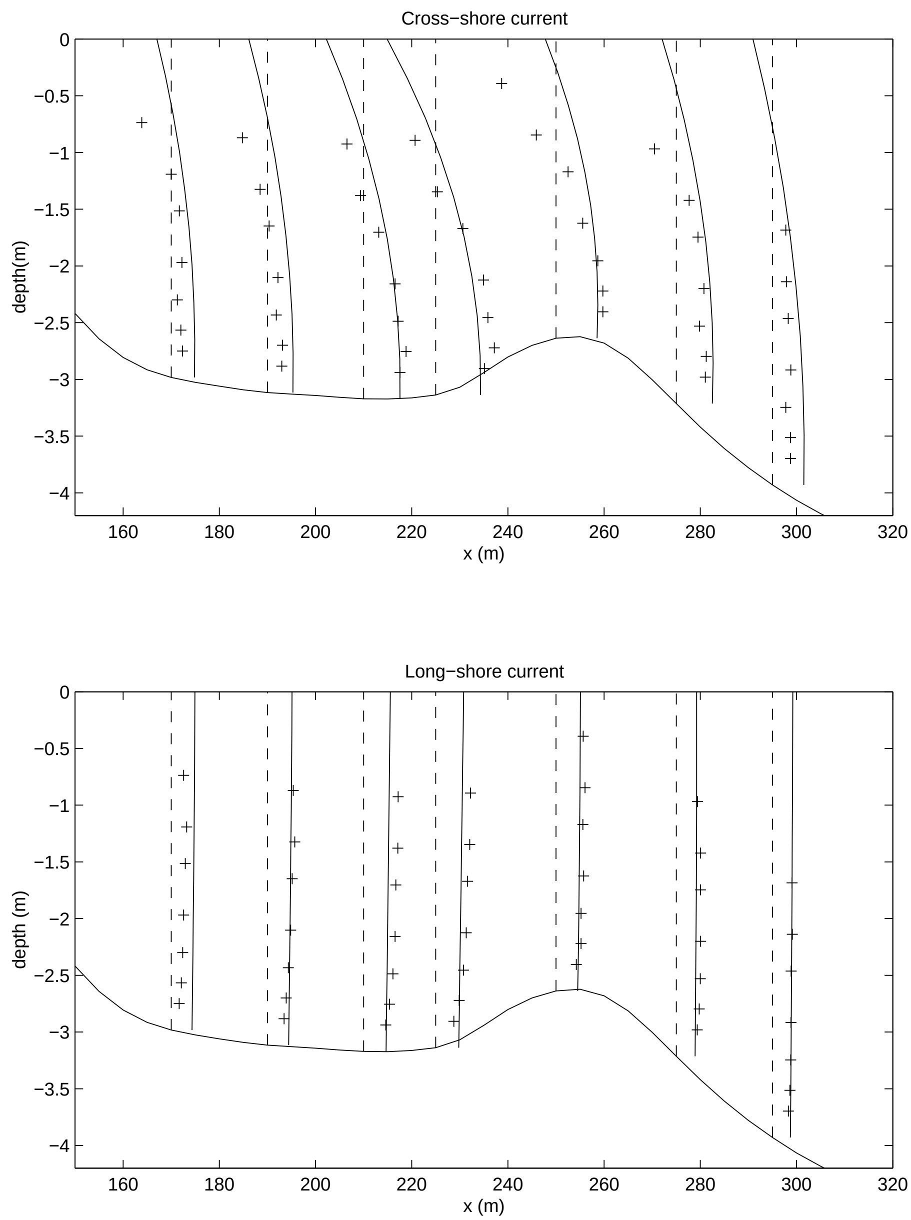

Figure 2: Comparison to Duck94 undertow measurements for October 12, 1994, 1230EST (high tide). Top: Cross-shore velocities. Bottom: Longshore velocities. Solid line: results from REF/DIF-S-forced SHORECIRC using Battjes and Janssen wave decay with Stive and deVriend roller. Crosses: data.

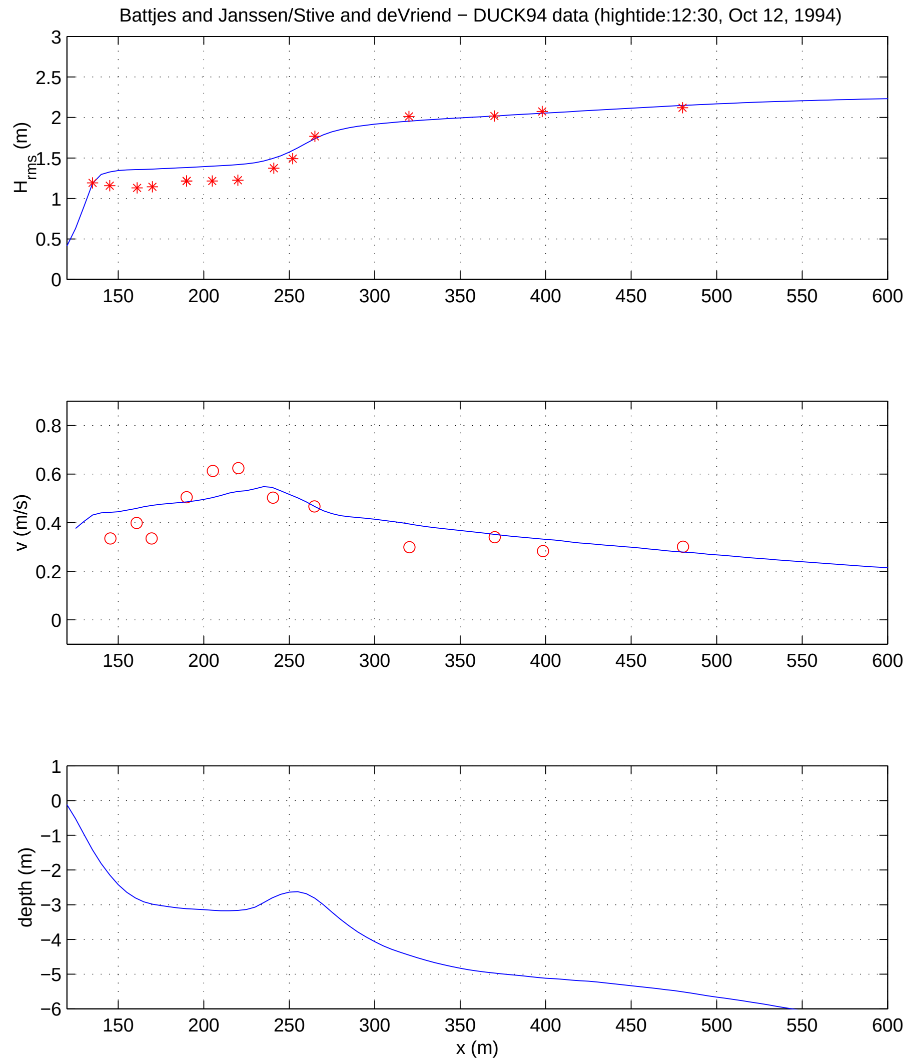

Figure 3: Comparison to Duck94 waveheight and longshore current measurements for October 12, 1994, 1230EST (high tide). Top: H_{rms}. Middle: Depth averaged velocity. Bottom: Depth profile. Solid line: results from REF/DIF-S-forced SHORECIRC using Battjes and Janssen decay with Stive and deVriend roller. Asterisks: waveheight data. Open circles: depth-averaged current data.

large scale wave propagation models such as SWAN (Booij et al. 1999). These models are typically phase-averaged, and thus some transformation of variables is required for input to the phase-resolving wave model REF/DIF-S. This problem becomes more involved if wave conditions along the offshore boundary of the nearshore grid change substantially.

Since the REF/DIF-S model is linear, the phase relationships between different spectral components are arbitrary; however, the spatial gradients of the phase for each component are essential since they relate to the wave angle. Given directional spectra at every point along the offshore boundary of the nearshore grid $S(f,\theta)(y)$, the transformation to complex ampitudes A_n is:

$$A_n(y) = \sqrt{2S(f,\theta)(y)\Delta f \Delta y}\, e^{i\int \lambda(f,\theta,y)dy} \tag{16}$$

where:

$$\lambda(f,\theta,y) = k(f,y)\sin\theta \tag{17}$$

and where the maximum value of n is the number of frequencies times the number of directions. This has the effect of phase-lagging the offshore wave condition for each frequency and direction component along the offshore boundary of the nearshore grid. In this manner the variation due to wave propagation offshore of the nearshore grid is properly reflected in the initial condition.

The modeling system is applied to the bathymetry of the upcoming Nearshore Canyon Experiment (NCEX), slated to begin in fall 2003 near La Jolla, California. The domain and waveheights from the SWAN model are shown in Figure 4; it is apparent that there is considerable variation in the nearshore wave climate. The bathymetry of the domain was gleaned from the National Ocean Survey (NOS) at 3 second resolution. The resolution of the SWAN simulation was $\Delta x = 38.5m$ and $\Delta y = 46.5m$, with the nearshore grid (shown in the small box in Figure 4) set at 16 times this resolution ($\Delta x = 2.40625m$, $\Delta y = 2.90625m$). Figure 5 shows the nearshore waveheights and depth-averaged circulation from the REF/DIF-S - SHORECIRC system. Though the bathymetry in this area is quite planar, severe bathymetric variations offshore of this area exacts a large degree of variation on the waveheight field, resulting in a circulation field more complex than the planar bathymetry would imply.

CONCLUSIONS

A coupled wave-hydrodynamic model system was described. This system is comprised of a random nearshore wave model (REF/DIF-S) coupled with a quasi-3D nearshore hydrodynamic model (SHORECIRC). Certain enhancements were made to the REF/DIF-S model to make it a more general, flexible model. To compliment the existing default wave dissipation mechanism of Thornton and Guza (1983), the waveheight decay formulation of Battjes and Janssen (1978) was included as an option. Random-wave formulations of roller-induced short wave volume flux and radiation stresses were developed and implemented into the REF/DIF-S model. These quantities are dependent on the wave roller description used. We detailed two mechanisms:

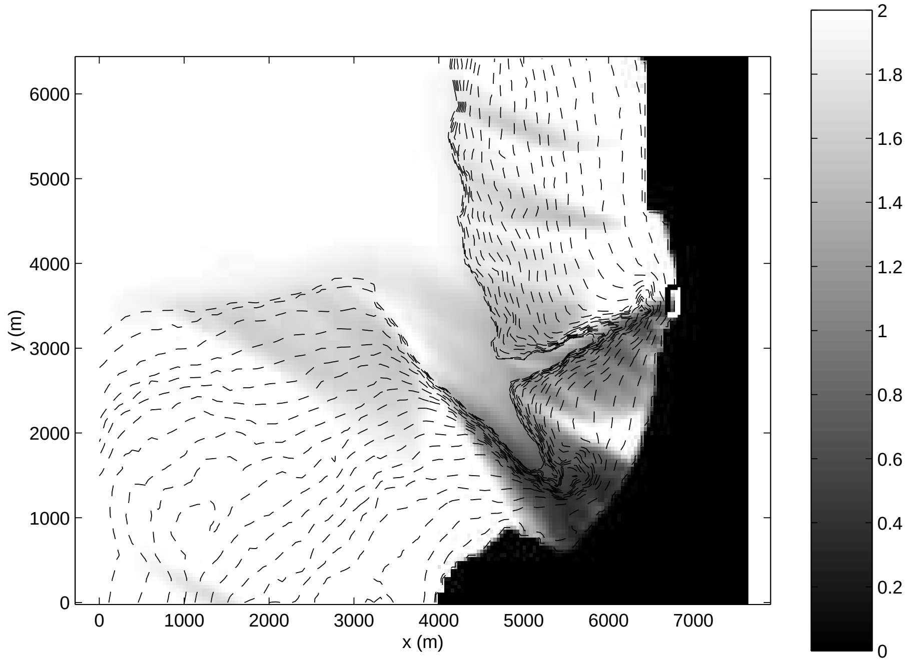

Figure 4: Bathymetry and $H_{1/3}$ over NCEX bathymetry, Scripps Canyon, CA. Nearshore grid outlined on right side of figure near coast.

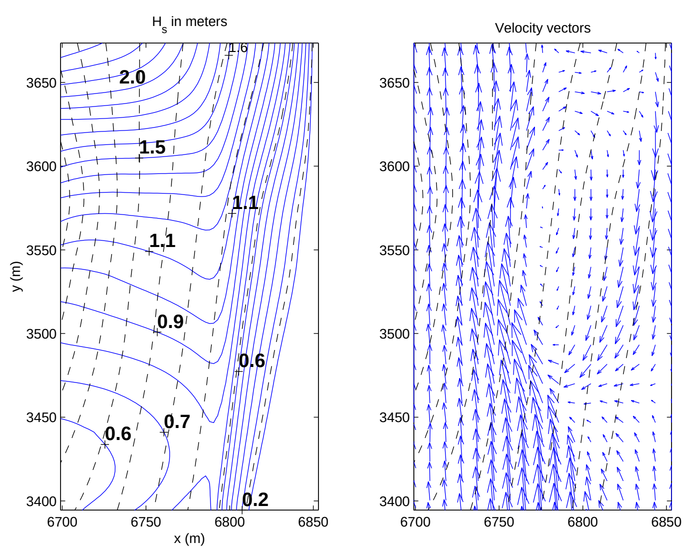

Figure 5: Waves and currents over nearshore grid, Scripps Canyon. Left: $H_{1/3}$. Right: Current vectors; the longest vector corresponds to 1 m/s.

a “static” roller whose properties are only based on local conditions, and the evolving roller of Stive and deVriend (1994). Additionally, wave energy dissipation was used to weight the surf-zone contribution to the overall eddy viscosity coefficient for use in modeling turbulent shear stresses.

Comparisons to laboratory data (Reniers and Battjes 1997) and field data from the Duck94 experiment revealed that evolving roller of Stive and deVriend (1994), along with the quasi-3D dispersive mixing in SHORECIRC, yields the best results. Deactivating the dispersive mixing leads to insufficient mixing and an unrealistically-narrow longshore current profile; use of the static roller yields the same results.

Finally, a capability for nesting the coupled modeling system inside a larger scale model was described. A method for transforming directional spectra saved along the offshore boundary of a nearshore grid to complex amplitudes was detailed. The system was applied to the site of the upcoming NCEX experiment; the offshore wave conditions were provided over the large scale region by the SWAN model (Booij et al. 1999). Directional spectra were saved along the offshore boundary of the nearshore grid and transformed into complex amplitudes for the REF/DIF-S model, the results of which were subsequently input to SHORECIRC. The resulting waveheights and currents over the nearshore domain showed considerable variation, despite the rather planar bathymetry in the nearshore.

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